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A Design of Experiments- Based Method for Point Selection in Approximating Output Distributions

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ABSTRACT

The goal of this research is to find a computationally efficient and easy to use alternative to current approximation or direct Monte Carlo methods for robust design. More specifically, a technique is sought to use selected deterministic analyses to obtain probability distributions for analyses with large inherent uncertainties.

Previous research by the authors has presented a promising class of methods known as Discrete Probability Matching Distributions (DPOMD). This paper introduces a new type of DPOMD better suited to problems with larger numbers of random variables. This new type utilizes a fractional factorial design of experiments array in combination with an inverse Hasofer-Lind standard normal space transform. The method defines points in the problem space that represent the moment characteristics of the input random variables.

This new method is compared to two other approximation techniques, Descriptive Sampling and Response Surface/Monte Carlo Simulation for three common aerospace analyses (Mass Properties and Sizing, Propulsion Analysis and Trajectory Simulation). A Monte Carlo analysis with corresponding error bands is used for reference. Preferences for probabilistic analysis each of these problems are determined based on the speed and accuracy of analysis. These results are presented here. The new DPOMD technique is shown to be advantageous in terms of speed and accuracy for two of the three problems tested.

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NOMENCLATURE

APAS	Aerodynamic Preliminary Analysis System
C_d	drag coefficient
C_L	lift coefficient
DPOMD	Discrete Probability Optimal Matching Distribution
DS	Descriptive Sampling
GLOW	gross liftoff weight
I_{sp}	specific impulse
ISS	International Space Station
KSC	Kennedy Space Center
MC	Monte Carlo
MR	Mass Ratio (liftoff mass / burnout mass)
OML	outer mold line
POST	Program to Optimize Simulated Trajectories
RSE	Response Surface Equation
SSDL	Space Systems Design Laboratory
SSTO	single stage to orbit
T_{vac}	vacuum thrust
μ	mean
σ	standard deviation

BACKGROUND

The demanding physics of launch vehicles creates aggressive performance requirements and often creates a temptation to design to the edge of feasibility. Because this can often lead to an unacceptably high probability of failure, launch vehicle design should have probabilistic information in its conceptual optimizations.

What is needed is an engineering method to efficiently and accurately calculate the uncertainty contained in a design analysis so that decisions can be made to minimize the impact of this uncertainty early in the design cycle, when such decisions are relatively cheap. This has the potential to provide critical design information.

This information goes unexpressed in traditional optimization and can result in a deterministically constrained optimum that will often lead to a high probability of an infeasible design (Ref. 1). This is due to the tendency of constrained optimums to lie directly on constraint boundaries. When uncertainty is added to these results, the probability will spread around the deterministic point. As a result, much of the probability will extend beyond the feasible frontier. This is shown in Fig. 1. Aerospace conceptual design therefore requires fast and accurate methods for determining system uncertainty.

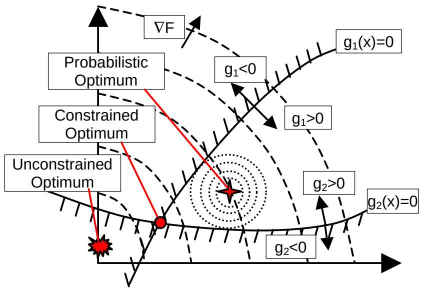

Figure 1 – Comparison of Constrained Optimums

Another advantage of probabilistic optimization is that it may be possible to create designs that are insensitive to sources of uncertainty. This is the idea of robust design, first introduced by Taguchi (Ref. 2). The intent of robust design is to use those factors over which the designer has control to achieve a result with low uncertainty in a response of interest. A great deal of work has been done recently to bring this idea into conceptual design using a variety of techniques. (Ref. 3) Through this work, the usefulness of this type of conceptual information has been shown using practical engineering approaches. It has laid the groundwork for probabilistic conceptual design and generated a great deal of interest in the subject among those in the aerospace industry.

The most general estimation technique is Monte Carlo simulation. It makes no assumptions approximating the analysis it is simulating nor does it require approximation of input distributions beyond that of random number generation. The major drawback of this technique is computational expense. To reduce this expense, several reduced expense methods have been proposed. To verify this cost savings as well as the accuracy of higher order methods, any reduced-expense sampling scheme should be compared to a Monte Carlo analysis in a test case. This means that these higher order methods are primarily useful for either probabilistic optimization, where repeated analyses of a similar solution space are required or on problems that have been proven to be compatible in previous research efforts.

One such method, Discrete Probability Optimal Matching Distributions (DPOMD), is presented in this paper. It utilizes a combination of design of experiments-based exploration and distribution space transforms to match the second order moments and interactions of an input distribution. It has been compared to a variety of methods (Refs. 4, 5), including Monte Carlo simulation, to determine its utility for a variety of common launch vehicle design problems. It is intended that this method be able to accurately predict the moments and interactions for problems with high dimensionality. In part, development of this technique was driven by the needs of a new probabilistic framework developed by the authors (Ref. 6).

RESEARCH GOALS AND OBJECTIVES

This research intends to demonstrate the use of DPOMD while at the same time showing its accuracy for several common aerospace vehicle analyses. The goals and objectives relating to this are as follows:

- A new member of a family of promising techniques, Discrete Probability Optimal Matching Distributions (DPOMD) for the probability prediction of a single analysis will be demonstrated.

The goal is to demonstrate the application of this new technique to launch vehicle conceptual design contributing analyses. This way, if the accuracy of the method is good, it can readily be used in subsequent launch vehicle conceptual studies. This application will also show the relative ease of setup for this method. The demonstrated ease of setup should reduce the opportunities for user error in application when compared to some other fast approximation techniques. It will increase the real-world accuracy and decrease the overall expense of the technique as it is applied after this research. This is partially due to the fact that human interface time is typically the most expensive aspect of any engineering enterprise.

To measure the success of the research with respect to the goal of demonstrating the new method, a detailed discussion of the setup of the method will be included. This should indicate how much effort and how many user assumptions were required. The objective here is that it be run in a very “black-box” manner, using information easily obtained by the user to generate quality uncertainty information for many types of problems.

- Several competing methods should be tested on several aerospace contributing analyses to find methods best suited for those analyses.

The objective for measuring the success or failure here will be the identification of a method for each of the analyses that is both fast and accurate. The knowledge gained by these tests should show the strengths and weaknesses of the probability analysis methods in terms of the contributing analysis being tested. More specifically, the goal for computational speed relates to its effect on the users ability to optimize the problem. Therefore, the goal here is for reduced computational expense expressed in actual function calls. A secondary goal related to these tests will be to gauge the accuracy and expense of the new DPOMD method for uncertainty analysis. This goal will be met during the course of the primary goal.

The uncertainty methods will be compared on the basis of their accuracy and their computational expense. Accuracy will be measured by the relative error of the relevant inputs and outputs when compared to a Monte Carlo simulation. Whether or not an input or output is considered relevant will be determined by the requirements made on the contributing analysis by other contributing analyses in a potential launch vehicle conceptual design framework. Preferred methods for each of these contributing analyses will be determined based on a combination of speed and accuracy.

The second objective for this comparison will be to measure the computational expense of the different methods. This will be measured by the time it takes to execute a single probabilistic analysis. At least a two order of magnitude improvement in computational expense is expected when the approximation methods are compared to a typical Monte Carlo analysis. This goal is essential to enabling optimization, since this process can entail hundreds of analyses of the overall system. Some methods included a great deal of up front computation to generate a metamodel, but then the metamodel was inexpensive to execute. For the sake of a simple comparison, the computational expense of the metamodel generation was excluded from the expense totals.

TECHNIQUES

Two types of DPOMD have been previously presented (Ref. 7). Both types relied on full factorial random variable space exploration to describe higher order input moments than are being described here. Because of the limited scalability of these full-factorial methods, a new DPOMD method was sought to more sparsely explore large input random variable spaces.

To this end, a method based on a two level fractional factorial experiment design was created. This technique is shown later to have good accuracy for several types of analyses while only growing linearly with the number of input variables. This means that problems with high dimensionality, such as mass properties and sizing (~40 random input variables), can be modeled using a relatively small number of runs.

Part of this method is based on a linear transform first proposed by Hasofer and Lind (Ref. 8). This takes joint multinormal variables in the problem space and transforms them into a standardized normal space. Here the joint multinormal can be expressed by independent standard normal distributions. The effect of this transform on discrete distributions is that it takes a discrete distribution with a certain mean vector and covariance matrix and expresses the points in a space where they have mean vector zero and an identity covariance matrix. DPOMD takes advantage of this transform by creating a discrete two-level fractional factorial distribution with zero mean and identity covariance matrix and performing an inverse Hasofer-Lind transform based on the desired mean and covariance matrix to create a discrete distribution in the real problem space with the desired second moment characteristics.

To create a DPOMD using this technique, a discrete distribution with zero mean vector and identity covariance matrix is required. This requirement is satisfied in the form of a fractional factorial two level design of experiments array with high and low levels set to –1 and +1 respectively. Because there are many options for creating fractional factorial arrays, it was assumed that the array with the highest possible resolution number would capture the most useful analysis response. An experiment's design resolution R is one where no n factor effect is confounded with another effect containing fewer than R – n factors (Ref. 9). For example, in a resolution IV design, the lowest order effect that can be confounded with a first order (one factor) is a third order (three factor) effect. As further example, second order effects can be confounded with other second order effects.

A Matlab© code was created using the direct generation method (Ref. 9) that creates maximum resolution fractional factorial designs. The direct generation method uses multiplications of different combinations of the basic factor columns in the design to create runs for factors that are non-basic factors. A basic factor is defined as a factor in a full factorial design that has size equal to the number of total factors minus the reduction. This smaller, full factorial design of main factors is then used generate additional columns to create a design with more factors. To create a design with the highest possible resolution, the largest combination of basic factors possible should be used to create settings for the new variables as is shown in Table 1.

This code cycles through all the combinations of basic factors, beginning with the longest (all factors) and progresses until all the columns for the desired non-basic factors have been created. The resultant array therefore has the desired properties of maximum resolution with the desired number of runs.

Table 1 – $2_{IV}^{(4-1)}$ Fractional Factorial Design Using Direct Generation Method

Run #	Basic Factor A	Basic Factor B	Basic Factor C	Factor D = A * B * C
1	-	-	-	-
2	-	-	+	+
3	-	+	-	+
4	-	+	+	-
5	+	-	-	+
6	+	-	+	-
7	+	+	-	-
8	+	+	+	+

Once the fractional factorial has been generated, an inverse Hasofer-Lind transform must be calculated. The Hasofer-Lind transform and its inverse are shown in Eqns. 1 and 2 (Ref. 8).

$$T_{trans}(\bar{x}-\bar{\mu}) = \bar{x}_{sns} \quad (1)$$

$$\bar{x} = T_{trans}{}^{-1}\bar{x}_{sns} + \bar{\mu} \quad (2)$$

Where x is a vector of positions in the analysis space, x_{sns} is a position vector in the standardized normal space, μ is a vector of means in the analysis space and T_{trans} is a transformation matrix defined by the relation in Eqn 3.

$$T_{trans} C_{\sigma} T_{trans}{}^{T} = I \quad (3)$$

The matrix T_{trans} is found by way of a Cholesky decomposition (Ref. 10) of the covariance matrix C_{σ}. Because C_{σ} is positive definite by definition, a Cholesky decomposition will yield the following:

$$\begin{aligned} R^T R &= C_{\sigma} \\ &= R^T I R \end{aligned} \quad (4)$$

so

$$(R^{-1})^T C_{\sigma} R^{-1} = I \therefore \quad (5)$$

$$T_{trans} = (R^{-1})^T \quad (6)$$

$$T_{trans}{}^{-1} = R^T \quad (7)$$

Where R is an upper triangular matrix. In the case of fractional factorial DPOMD, the required matrix is T_{trans}^{-1}, so no matrix inversion is required after the Cholesky decomposition.

The fractional factorial method matches the mean vector and covariance matrix of the inputs. It is limited in that it cannot account for higher than second order expectation values. This means that input distributions with high skewness will be missing this characteristic in the fractional factorial discrete probability distribution model. It also has the inherent assumption that the output response can be characterized using a fractional factorial output response.

The primary advantage of this method compared to other DPOMD techniques is its scalability. While the number of sample points can be specified by the user to a degree, the sample point count must be larger than the number of variables if the set is to span all the dimensions of the problem. If the number of sample points is less, not all the variables will be considered. To compound this, the ignored variable has not been determined to be negligible to the response by any kind of screening process.

TEST PROCEDURE

Because this paper anticipates that different distribution estimation techniques will be better suited to different analyses, each was tested using several probabilistic techniques to determine which best analyzes the characteristics of the particular problem. The three analyses examined are, mass properties/sizing, trajectory and propulsion. The following section provides the rationale for which methods were tested, briefly describes the tests performed on each analysis and presents the results generated.

It is hoped that these tests will indicate which of the methods for probability distribution approximation is the best suited for these types of launch vehicle contributing analyses. By using a single analysis, a costly Monte Carlo analysis must only be run once as a reference for the other approximation methods. For the sake of practicality, the general techniques tested on each analysis will be the same.

Because the goal of using these approximation methods is to enable optimization, the test requires methods that can be executed with a reasonable number of samples, with the number of samples depending on the analysis computational requirements. Reasonable for a contributing analysis is considered to be around a minute. This allows for optimizations of the system on the order of one day. Next, the method must provide correlations for output variables that are either coupled to other outputs. This ensures complete expression of the outputs and allows for a probabilistic multiobjective formulation similar to those presented by Bandte, et al. (Ref. 11).

Approximation methods that provide this type of information fall into two categories, variance reduction and metamodeling methods. These include among others importance-based sampling, Latin hypercube sampling, control variates, antithetical variates, discrete probability optimal matching distributions (DPOMD) and descriptive sampling The meta-modeling method considered here is a response surface with variable screening.

To be more specific, the response surface procedure first uses a screening array to reduce the number of variables considered to 10, then fits a quadratic response surface equation to a higher order design of experiments array for the 10 variables.

Importance-based sampling was eliminated from consideration because it is primarily useful for accelerating the calculation of constraint comparisons. Because information about the whole distribution of outputs is desired and the only constraint comparisons were considered to an 80% confidence level, this method was not expected to provide much benefit. Latin hypercube and stratified sampling were not considered due to their similarity to descriptive sampling. In addition to this, Saliby (Ref. 12) has shown descriptive sampling to converge more quickly than Latin hypercube, while stratified sampling seems not as popular as other methods, judging by the relative scarcity of the available literature. Control Variates were left out because of the need for user assumptions about the output variables. This violates the “black box” goal of the test. Finally, Antithetical Variates were not studied because of their only slight promised performance benefit when compared to Monte Carlo simulation (Ref. 13).

This left Monte Carlo simulation, DPOMD, descriptive sampling and response surface methods as the candidate methods. The tests of these methods and the particular types of each chosen method used is described in the following sections. They are presented in groups defined by the contributing analysis being tested.

The probabilistic methods described above are compared using three common launch vehicle conceptual design analyses. These are described in the following three sections.

Mass Properties and Sizing Test

The platform for the mass estimating relationships is a Microsoft Excel© spreadsheet containing an iterative set of mass and geometry estimating relationships. If this were a deterministic analysis, the difference between the mass ratio calculated as required by the trajectory and the available mass ratio calculated by the sizing analysis would be driven to zero by the Excel© Solver add-in through altering the vehicle size. A corresponding weight breakdown structure would be the output of this analysis.

For probabilistic sizing, there are several options for sizing. One method would be to allow the length of the vehicle to float with every trial, ensuring that every predicted scenario is a closed vehicle. This methods presents difficulties for proceeding to detailed design, as there is no single size to which components can be designed. This makes the vehicle size a far-term variable to be determined at a later date. To do this would ensure that all trials could be sized, but it would also prevent a decision on the Outer Mold Line (OML) size from being made by the current analysis. This is essentially a probabilistic analysis wrapped around the existing sizing algorithm.

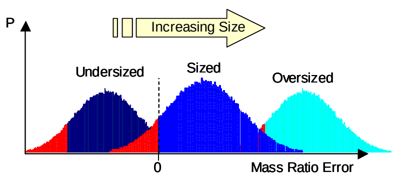

Figure 2 - Probabilistic Sizing

For the current research, a decision on OML size is desired at the conceptual stage. Therefore, this quantity is a near term variable and will be set so that it meets the propellant requirements of the scenarios to a certain confidence level. This means that the mass ratio error for a particular size vehicle is not a single value, but an entire distribution of errors. This is the performance constraint that must then be met. This is done by altering the vehicle size until a desired percentage of the mass ratio errors are driven to positive values. This problem is equivalent to ensuring that the required mass ratio is lower than the available mass ratio to a certain confidence level. Fig. 2 illustrates this. The input and output characteristics of the analysis can be found in Table 2.

Table 2– Input and Output Properties

Variable Type	Number
Correlated Normal Inputs	3
Independent Triangular Inputs	35
Correlated Outputs	30

Mass Properties and Sizing Results

The aim of this test was to determine the characteristics of several methods of uncertainty approximation when applied to a conceptual launch vehicle mass properties analysis and then select a preferred method.

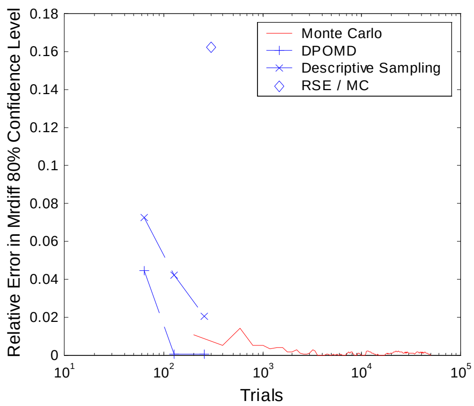

Figure 3 – Trial History of MR Confidence Level

The first goal has been met. The characteristics of this problem are shown with results for response surface, discrete probability optimal matching distribution and descriptive sampling methods.

Of the techniques tested, the DPOMD method for the two higher sample sizes was able to best match the Monte Carlo simulation for the output parameters. While the descriptive sampling method was close for some parameters, the DPOMD was clearly the method of choice for this contributing analysis. This can be seen in Fig. 3, which shows a trial history of the accuracy of all of the approximation methods. The Monte Carlo simulation accuracy was calculated like the others by comparing it to the final answer generated by the Monte Carlo.

Table 3 – Mass Properties Results Comparison

	GLOW μ	GLOW σ	G/D corr.	Drywt. μ	Drywt. σ	MR 80% c.l.
Monte Carlo	1,681,950 lb.	34,280 lb.	44.2 %	143,630 lb.	3,760 lb.	0.981
95% c. l.	± 300 lb.	± 210 lb.	± 0.7%	± 33 lb.	± 20 lb.	± 0.03 %
RSE / Monte Carlo	1,686,275 lb.	34,220 lb.	50.4 %	147,000 lb.	3,300 lb.	1.140
Abs. Rel. error	0.257 %	0.175 %	14.0 %	2.35%	12.2 %	16.2 %
64 run DPOMD	1,681,960 lb.	32,625 lb.	-7.27 %	143,640 lb.	3,290 lb.	0.937
Abs. Rel. error	0.000860 %	4.84 %	116 %	0.00514 %	12.5 %	4.45 %
128 run DPOMD	1,681,970 lb.	34,275 lb.	44.2 %	143,645 lb.	3,770 lb.	0.980
Abs. Rel. error	0.00121 %	0.0262 %	0.110 %	0.00875 %	0.392 %	0.079 %
256 run DPOMD	1,681,970 lb.	34,201 lb.	44.2 %	143,644 lb.	3,750 lb.	0.982
Abs. Rel. Error	0.00120 %	0.240 %	0.124 %	0.00868 %	0.242 %	0.075 %
50 run DS	1,681,967 lb.	34,118 lb.	46.8 %	143,648 lb.	3,695 lb.	0.910
Abs. Rel. error	0.001 %	0.483 %	5.79 %	0.0117 %	1.67 %	7.23 %
100 run DS	1,681,952 lb.	33,698 lb.	51.4 %	143,641 lb.	3,710 lb.	0.940
Abs. Rel. error	0.0001 %	1.71 %	16.2 %	0.00685 %	1.26 %	4.21 %
200 run DS	1,681,982 lb.	35,086 lb.	53.2 %	143,650 lb.	3,701 lb.	0.961
Abs. Rel. Error	0.00192 %	2.34 %	20.2 %	0.0111 %	1.50 %	2.06 %

Table 3 and Fig. 3 indicate that the RSE/MC method was somewhat inaccurate. While some the parameters not shown in Table 3 were accurately predicted by the response surface, the errors on certain variables were too high to allow the use of this type of simulation. On the Excel© platform, the time to evaluate the RSE for 50,000 trials was 12.8 minutes on a Pentium III 850 MhZ computer. The time to evaluate the same number of Monte Carlo trials was 14.1 minutes. If executed using a custom C++ Monte Carlo RSE evaluation program, this cost is much smaller, taking only 5.5 seconds on an SGI Octane workstation.

The approximation methods that took on the order of one hundred trials were even quicker still. While not as accurate, the 64 trial methods completed in 1.1 seconds, the 128 trial methods in 2.2 and the 256 methods in 4.4 seconds, on the order of the expense of using the RSE on the workstation. A complete listing of the computational expenses can be found in Table 4.

Table 4 – Mass Properties Execution Times

Number of Trials	Platform	Time
50,000 Excel© Analysis Calls	Pentium III PC	14.1 min.
50,000 Excel© RSE	Pentium III PC	12.8 min.
50,000 C++ RSE	SGI Octane	5.5 sec.
256 Excel© Analysis Calls	Pentium III PC	4.3 sec.
200 Excel© Analysis Calls	Pentium III PC	3.4 sec.
128 Excel© Analysis Calls	Pentium III PC	2.2 sec.
100 Excel© Analysis Calls	Pentium III PC	1.7 sec.
64 Excel© Analysis Calls	Pentium III PC	1.1 sec.
50 Excel© Analysis Calls	Pentium III PC	0.85 sec.

Interpretation of the results presented here show that the choice method for this analysis, with this set of inputs and outputs, was the fractional factorial DPOMD. More specifically, the 256 trial version was selected due to the fact that it was accurate for all the selected parameters of interest, and still had one of the lowest computation times. While the descriptive sampling method does not seem to depend as much on sample size, the accuracy of the higher order DPOMD methods is higher.

Propulsion Analysis Test

The propulsion analysis considered here consists of a deterministic rocket engine design analysis tool, SCORES (Ref. 14), with an uncertainty approximation method wrapped around the outside. SCORES was developed in the Space Systems Design Laboratory (SSDL) by Way to provide quick, conceptual-level estimates of rocket engine performance, taking into account such factors as chemical equilibrium and nozzle type, but does not require design information about powerhead pump systems.

SCORES consists of two analyses, the first of which calculates the equilibrium chemical state of the propellants in the combustion chamber and the second of which does a frozen flow, converging-diverging nozzle calculation to generate the ideal thrust of the proposed engine. Regressed efficiencies are then placed on the nozzle and combustion chamber depending on the type of engine cycle selected by the user.

To size the rocket, SCORES uses a simple scaling (Ref. 14) algorithm. To create an engine of a specified thrust, first output parameters for a baseline engine with a throat area of one square inch are calculated, then the throat area of the baseline engine is changed linearly with the reference thrust to create a new engine. This allows for rocket engine performance estimates commensurate with the amount of information available about the engine at this stage of development. It also allows SCORES to generate a sized engine with virtually no computational expense beyond that of a single rocket analysis.

For this particular simulation, four inputs were selected as noise variables. They were the gross liftoff weight, the combustion chamber pressure, the engine power-to-weight ratio and the mass ratio required. The mass ratio required was included in the test because the correlations between it and the propulsion output variables are required by the mass properties and sizing analysis. In addition, there were three deterministic variables used as inputs to the analysis. These were vehicle thrust-to-weight ratio at liftoff, engine nozzle expansion ratio and propellant mixture ratio.

Propulsion Test Results

The goal of this section was to provide a fast and accurate method for estimating the output distribution information of a rocket engine design simulation. This goal has been met in that several methods showed low error in estimating the probability of several parameters for the propulsion contributing analysis.

Another goal was to measure the error in approximating the probabilistic analysis using the best possible method. The chosen method, DPOMD, showed minimal errors on all the output parameters of interest. There should not be any problems when applying this method to the propulsion contributing analysis.

The relative error history in Fig. 4 shows why the response surface method was eliminated from contention for use in the full system-level optimization problem. It’s error in estimating the very important engine efficiency standard deviation was far too high. The other methods had errors an order of magnitude lower for this parameter.

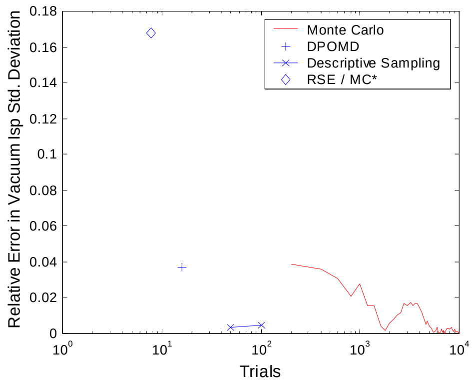

Figure 4 – I_{sp} Std. Dev. History

Fig. 5 illustrates the difficulty the otherwise accurate and efficient descriptive sampling method had with predicting some of the correlation coefficients in this research.

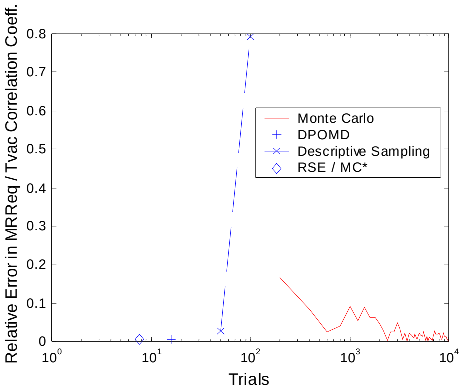

Figure 5 – Example Correlation History

The execution times of the above methods are hinted at in the previous figures. These times varied greatly from one method to the other, as evidenced by the use of a log scale in the summary figures. The amount of computational time required for the runs for each method is given in Table 4.

Table 4 – Propulsion Execution Times

Method	Trials	Platform	Time
Monte Carlo	10,000 SCORES calls	SGI Octane	24 min.
Descriptive Sampling	100 SCORES calls	SGI Octane	14 sec.
Descriptive Sampling	50 SCORES calls	SGI Octane	7 sec.
DPOMD	16 SCORES calls	SGI Octane	2.3 sec.
RSE / MC	10,000 C++ RSE calls	SGI Octane	1.1 sec

Trajectory Analysis Test

The trajectory analysis for this test is a single stage to orbit (SSTO) trajectory utilizing the POST (Ref. 15) trajectory optimization program. It takes a set of pitch control variables at selected points along the trajectories and optimizes these for a user specified objective, usually the maximization of burnout weight, while at the same time meeting constraints based on desired burnout conditions, usually a target orbit. This idea is illustrated in Fig. 6.

This is a deterministic analysis when taken alone. However, with the inclusion of the GRAM99 (Ref. 16) atmosphere model, the trajectory analysis included a larger number of noise variables than either of the previous analyses. This model used regressed atmosphere data for selected launch ranges around the world to predict atmospheric conditions given certain known factors. For this Monte Carlo simulation, the known factors were considered random within a reasonable set of ranges. For the other analyses, because GRAM99 is an inherently stochastic program with internal random number generation, the results of a Monte Carlo simulation of just GRAM99 had to be fit at certain key points in the atmosphere in order to use the listed approximation methods. These key points were selected by visual inspection of the mean results for the GRAM99 Monte Carlo for the atmospheric parameters important to the physics of the trajectory simulation.

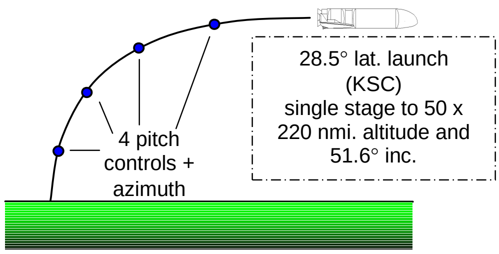

Figure 6 – Trajectory Analysis Overview

Once the deterministic version of the stochastic problem was formulated, several candidate methods for probabilistic analysis were tested at acceptable levels of computational expense. These candidate methods were the same as those in the previous tests. They were response surface / Monte Carlo analysis, descriptive sampling and discrete probability optimal matching distributions. These methods were tested for speed and accuracy on selected noise, coupling and output variables based on the goals and objectives stated earlier.

While there are many possible outputs from trajectory into other contributing analyses, experience with this contributing analysis for this particular system problem indicates that the only output variable absolutely necessary is the mass ratio required to make the target orbit. In order to test the methods' ability to estimate correlations, the correlation of one of the outputs to one of the inputs is also of interest.

While there are many possible outputs from trajectory into other contributing analyses, experience with this contributing analysis indicates that the only output variable typically necessary is the mass ratio required to make the target orbit. However, due to the unique requirements of the probabilistic multidisciplinary design framework, the correlation of one of the outputs to one of the inputs was also required. This is because the input gross liftoff weight (GLOW) and the output mass ratio required are both inputs to the propulsion contributing analysis.

The inputs for the analysis include all the information required to perform the analysis and optimization of the trajectory. This translates to information that must be given the GRAM99 (Ref. 16) in order to generate a random atmosphere deck. Also, for the test, the vehicle outer mold line (OML), including the engine exit area was assumed to be constant. This list of input noise variables is therefore as follows:

- Vehicle gross liftoff weight (GLOW) – This gives the trajectory analysis a necessary initial condition to start its trajectory integration.
- Vehicle thrust at vacuum condition (Tvac) – For a bell nozzle liquid rocket engine, this parameter, combined with total engine exit area gave the trajectory information about the thrust of the engine through a range of altitude conditions.
- Engine Specific Impulse at vacuum condition (Isp_{vac}) – This describes the propellant efficiency of the engine.
- Multiplier on C_l (Cl_mult) – This variable changes the overall aerodynamic lift coefficient at all conditions to simulate errors in the aerodynamic modeling.
- Multiplier on C_d (Cd_mult) – This changes the overall aerodynamic drag coefficient at all conditions again to simulate errors in aerodynamic modeling.
- Year of launch (year) – Information required by GRAM99 in order to generate accurate atmospheric condition scenarios.
- Month of launch (month) – Information required by GRAM99 in order to generate accurate atmospheric condition scenarios.
- Day of launch (day) – Information required by GRAM99 in order to generate accurate atmospheric condition scenarios.
- Hour of launch (hour) – Information required by GRAM99 in order to generate accurate atmospheric condition scenarios.

The multipliers on coefficients of drag and lift Cd and Cl are to account of errors in the aerodynamic dataset. The program APAS (Ref. 17) was used to generate the aerodynamic datasets for the OML of the launch vehicle used. It used a vortex panel method for subsonic and supersonic calculations and impact methods for hypersonic aerodynamics. The error for this code has been informally rumored to be about +/- 10%. However, as with any aerodynamic prediction, huge errors are possible from relatively small-scale phenomena. The other noise inputs are either probabilistic coupling variables to other contributing analyses, or inherently unknown quantities, like the time of launch.

It is important to note that these inputs were only used in the direct Monte Carlo trajectory simulation. The other analyses required parameterization of the atmospheric inputs. This yielded a different and much larger set of inputs for the approximation methods. These sets will be described later in the next section

Atmospheric Approximation

The approximation methods for this analysis could not be used on the same analysis as the Monte Carlo simulation due to the inherent randomness of the GRAM99 atmospheric model. Therefore, a multivariate normal distribution was fit to the data at selected points for each atmospheric parameter. The results were the same points used for the trajectory in the earlier Monte Carlo simulation.

Because the number of atmosphere data points (204) was too high to effectively use an uncertainty approximation method, the number of atmosphere points had to be reduced. The points were reduced by visually inspecting the mean values of the results of a Monte Carlo simulation performed on the GRAM99 atmosphere model. Only the points necessary to satisfy the trends of the mean atmospheric parameters were retained. This section reviews this selection process.

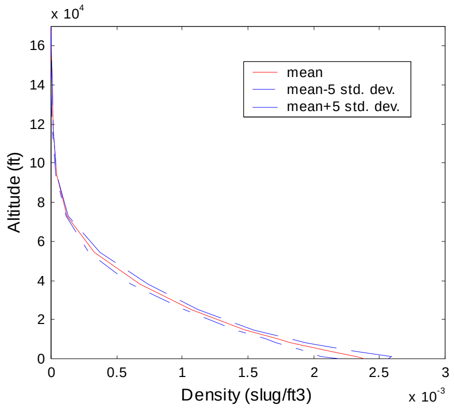

Figure 7 – Density v. Altitude with Uncertainty Bands

The means of the distributions at each of the points could be used as the criteria because of the relatively small perturbations from the means due to randomness. Fig. 7 shows uncertainty bands of plus and minus 5 standard deviations for air density. This is the number of standard deviations required to actually see the bands on the plot.

The parameters required by the trajectory analysis as a function of altitude were the following:

- Pressure – Ambient atmospheric pressure in lb./ft.^{2}
- Density – Ambient atmospheric density in slug/ft.^{3}
- Temperature – Ambient temperature in °R.
- Northern wind – wind velocity in the northerly direction in fps.
- Eastern wind – wind velocity in the easterly direction in fps.
- Vertical wind – downward wind velocity in fps.

These factors were calculated for 10,000 random trials based on random inputs the same as those used in the combined trajectory / GRAM99 Monte Carlo simulation performed in the previous section. A single multivariate normal distribution was then fit to all 204 output responses. After that, the means of each dimension of this multivariate normal distribution were plotted and points for the reduced atmosphere model selected using these plots.

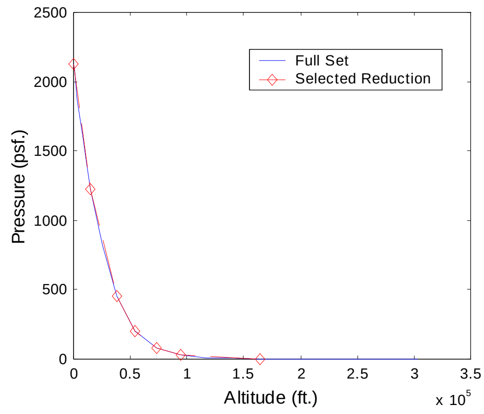

Figure 8 – Pressure Points Selected

The plot for mean pressure in Fig. 8 shows an exponential decay, as is to be expected. The dotted line shows the linear interpolation between the points. This seven point interpolation is nearly indistinguishable from the thirty four point version shown by the solid line. The final pressure on the interpolation table at an altitude of 210,000 ft. was assumed to be zero.

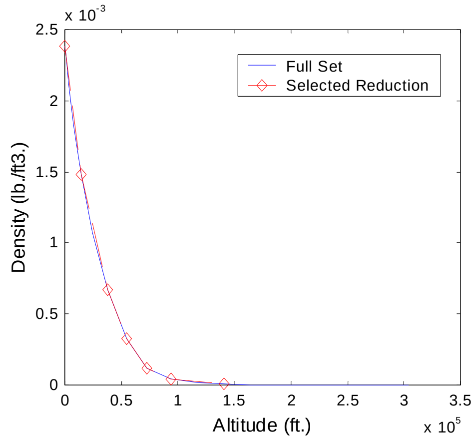

Figure 9 – Density Points Selected

The plot for density in Fig. 9 shows similar behavior to the pressure plot, therefore points were taken at similar altitudes. Again, the seven points approximation is hard to distinguish from the 34 point version. The final density of zero was also placed at 210,000 ft., just like the pressure table.

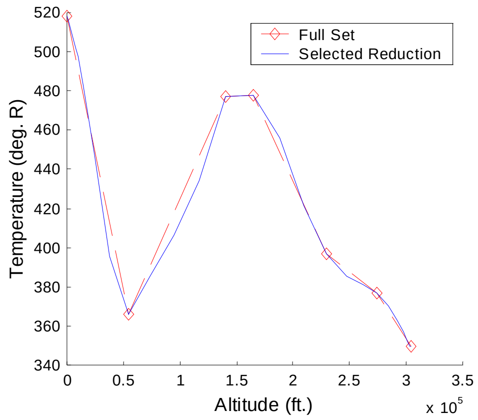

Figure 10 – Temperature Points Selected

Fig. 10 shows atmospheric temperature. It proved slightly more difficult to fit, but still is well represented by the reduced point table. For this table, because the temperature does not ever reach zero, the final value for the table was selected at an altitude of 304,000 ft. and was assumed to have the distribution described by GRAM99. It is important to note that the POST trajectory analysis was set to no extrapolation for all input tables, so this temperature would not be changed above this altitude during any of the trajectory simulation trials.

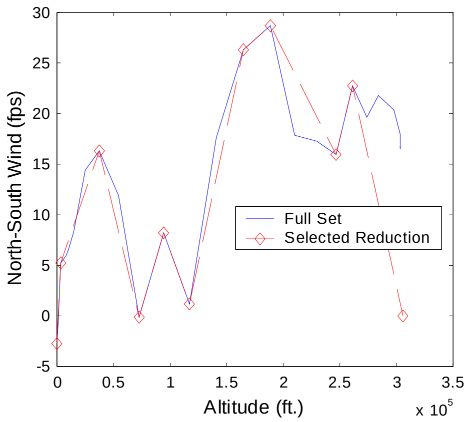

Figure 11 – Northerly Wind Points Selected

Because the northerly wind (Fig. 11) was a little more erratic than the previous parameters, more points were used to reduce the dataset. In this case, eleven points were selected at various altitudes, representing the changes in direction and magnitude for this particular wind component. While the fit of the linear interpolation seemed to fail at higher altitudes, it should be noted that the air density was assumed to be zero above 210,000 ft. This means that these points would not have had any force effect on the vehicle.

The fit on the east wind component in Fig. 12 using the reduced number of points was quite good. This is important as it is expected to be the most influential of the wind components, because of the direction of the launch and the expected magnitude of the wind. Changes in the body axis velocity would be expected to have the biggest effect on the overall trajectory performance. Seven points were used to represent this quantity.

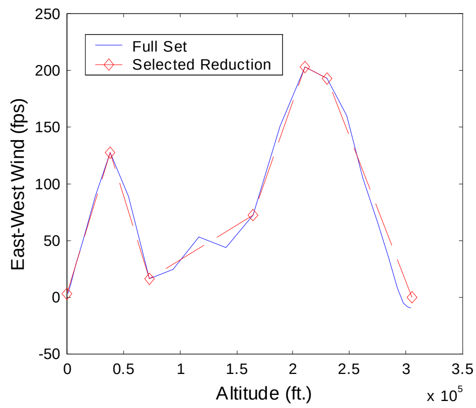

Figure 12 – Easterly Wind Points

While the vertical wind was the most erratic and least accurately fit of the parameters, the magnitude of the vertical wind components were small at low altitudes. This where they would have had the best chance to have an effect on the vehicle, so this was the area best represented by the reduced point set. At higher altitudes, there are upward wind components, but these were so high as to be unlikely to have an effect on the trajectory. Eight points were used to represent this physical effect.

The selection process presented here left the trajectory contributing analysis with 49 total variables, 5 that go directly into the trajectory analysis and 44 variables used to describe the atmospheric conditions. This means that the trajectory contributing analysis is now ready for the application of uncertainty approximation methods.

Trajectory Analysis Test Results

The primary aim of this analysis test is to find one or more uncertainty analysis techniques for trajectory optimization that are both fast and accurate. The accuracies of the methods are compared to a Monte Carlo simulation to then determine which candidate method would be used in the full, multidisciplinary optimization problem.

The first goal of testing the accuracies of the methods has been met. All the methods were tested at their most reasonable computational expense and the results are presented in the preceding sections.

Table 5 – Trajectory Analysis Results

	MRreq μ	MRreq σ	Mrreq/GLOW corr.
Monte Carlo	7.8934	0.0328	15.9%
95% c.l.	± 0.00064	± 0.00045	± 1.9%
CC RSE / MC	7.9436	0.0343	13.7 %
Abs. Rel. Error	0.637 %	4.54 %	13.7 %
DO RSE / MC	8.0088	0.0350	16.6 %
Abs. Rel. Error	1.46 %	6.78 %	4.47 %
DPOMD	7.9168	0.0343	15.3 %
Abs. Rel. Error	0.297 %	4.52 %	3.95 %
Descriptive Sampling	7.9170	0.0342	16.7 %
Abs. Rel. Error	0.299 %	4.25 %	4.89 %

Of the approximation methods tested, the descriptive sampling simulation technique proved to be the most accurate. However, the central composite response surface equation methods did not have as large errors during this test as in the previous ones. Combined with the significant cost savings involved with using this type of analysis as shown in Table 6, this is likely the preferred method.

The trial histories presented in Fig. 13 surprisingly show that the Monte Carlo analysis got closer to its final answer sooner than either of the sampling methods. This means that there would be almost no reason to use either of these methods, as the more general Monte Carlo simulation supercedes both the descriptive sampling and the discrete probability optimal matching distributions at their chosen resolutions for every output parameter of interest but MR-GLOW correlation coefficient.

Table 6 – Trajectory Analysis Execution Times

Method	Trials	Platform	Time
Monte Carlo	10,000 POST Runs	2 SGI Octanes	5 days
DPOMD	128 POST Runs	SGI Octane	3 hrs.
Descriptive Sampling	100 POST Runs	SGI Octane	2.5 hrs.
CC, D-Optimal RSE / MC	10,000 C++ RSE Calls	SGI Octane	1.1 sec.

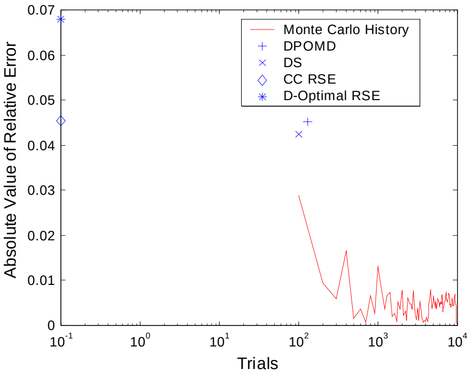

Figure 13 – History of MR Required Standard Deviation Error

The accuracy of all the approximation methods was slightly lower for this analysis test than in the others. A likely cause of this was the reduction of the atmosphere model described earlier in the section. While necessary due to the program setup of the GRAM99 tool, this nonetheless created an inherent difference in the problem statement for all the approximation methods when compared to the Monte Carlo simulation. The consistent error across all the analyses supports this. Compared to similar tests (Ref. 3) that neglected atmospheric uncertainty, the accuracy of approximation of this trajectory optimization problem was generally not as good.

CONCLUSIONS

- A new method of engineering uncertainty analysis, DPOMD, was demonstrated and tested. This was evidenced by a detailed description of the procedure, and a series of applications in the analysis testing section of this research.

- This new method was shown to have ease of setup, as the only inputs to the process were the moment information and the reduction factor for the fractional factorial design. This showed that the technique could be easily applied once the underlying algorithm had been programmed.

- Several techniques for uncertainty analysis on the conceptual launch vehicle design contributing analyses were tested and preferred methods were identified.

The tests revealed several things about the methods that were tested with respect to each of the contributing analyses. First, it is important that all of the output parameters were accurately represented by the methods. Some of the eliminated techniques were excellent on the main output parameters, but were far away from the reference for many of the correlations.

- Several sources for uncertainty were identified and incorporated into three common reusable launch vehicle conceptual design analyses.

These sources included weight, engine performance and atmospheric uncertainties. For the distributions available in the open literature, historical values were set. Otherwise, reasonable assumptions were made based on deltas around deterministic values. All major assumptions for this launch vehicle problem were expressed as noise distributions.

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